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What Do Fluctuation Measurements Say About $D = \gamma/k^2$ and Shear Stabilization of Drift Waves,

gamma/k-squared

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This note assesses recent measurements of fluctuations in experiments and numerical simulations as they concern $D = \gamma/k^2$ and shear stabilization of drift waves.

gamma/k-squared

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WHAT DO FLUCTUATION MEASUREMENTS SAY ABOUT $D = \gamma/k^2$

AND SHEAR STABILIZATION OF DRIFT WAVES?

I. Fluctuation Measurements Concerning $D = \gamma/k^2$

A. Introduction

The well-known estimate of the turbulent diffusion coefficient

$$D \leq \frac{\gamma_k}{k_x^2}\ , \tag{1}$$

(where γ_k is the growth rate and $\underline{k}$ is the wave number of the unstable waves) can be derived in a variety of ways. One way is to assume that the waves cause diffusion, add a term $- D\nabla^2 n$ to the ion continuity equation and adjust D so as to produce marginal stability. The theories of Dupree[1], Weinstock[2] and others[3] might be regarded as more sophisticated versions of this procedure.

However, the most popular derivation of Eq. (1) is that of Kadomtsev[4]. This begins by assuming that non-linear effects saturate the growth of the instability when

$$k_x^2 \langle \tilde{n}^2 \rangle \leq \left| \frac{dn_o}{dx} \right|^2 . \tag{2}$$

(Simple slab geometry is assumed with macroscopic quantities varying in the x direction. n_o is the mean and $\tilde{n}$ the fluctuating ion density.)

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From a quasi-linear treatment of the ion continuity equation one obtains for the particle flux

$$\langle \tilde{n}\, \tilde{u}_x \rangle \approx \frac{-\gamma \langle \tilde{n}^2 \rangle}{\frac{dn_o}{dx}} \quad . \tag{3}$$

If the estimate (2) for $\langle \tilde{n}^2 \rangle$ is now used in Eq. (3) (notwithstanding the inconsistency of this procedure) Eq. (1) follows. This estimate is the basis of many numerical transport calculations[5].

Clearly, the above argument is not entirely convincing and counter-arguments have been given[6]. Also Dupree and Tetreault[7] have recently reexamined the basis of the strong turbulence theory. Their conclusion is that $D \sim \gamma/k^2$ can only be valid if the plasma is very weakly unstable. (Their condition for stabilization by this mechanism is $\gamma/\omega < (k_{\parallel} v_i/\omega)^2$, where $k_{\parallel}$ is the wave number parallel to $\underline{B}$ and v_i is the ion thermal velocity.) Nevertheless it is widely believed that there is a comparatively simple relationship between diffusion and fluctuation levels, that Eq. (1) gives an upper limit to the turbulent diffusion, and that Eq. (2) gives an upper limit to the level of density fluctuations. Evidence can be adduced in support of these beliefs[8,9]. In the next section we show that there is also evidence against them.

B. <u>Evidence</u>

It is convenient to rewrite Eq. (2) in the form

$$\frac{\tilde{n}}{n_o} \approx (k_x L_n)^{-1} \ , \tag{4}$$

where L_n is the density scale length. We now discuss some experimental evidence.

(a) Okabayashi and Arunasalam[10] report studies of drift-wave turbulence in the FM-1 spherator. In their low shear case, with the plasma in a state of strong turbulence, they found the fluctuation level to be

$$\frac{\tilde{n}}{n_o} \sim 0.15. \tag{5}$$

For $(k_x L_n)^{-1}$ ($(k_\psi a)^{-1}$ in their notation) they found

$$(k_x L_n)^{-1} \sim 0.02 \text{ to } 0.03, \tag{6}$$

so the fluctuation amplitude is five to seven times larger than the estimate of Eq. (4). This translates (using Kadomtsev's prescription) into a diffusion coefficient of between 25 and 50 times larger than given in Eq. (1).

(b) Navratil and Post[11] observed large amplitude density fluctuations in a levitated octupole which they interpret as due to drift waves. For the parameters of this experiment (Helium plasma, $B = 750G$, $T_i = 0.2$ ev, $k_\perp \rho_i \sim 0.3$) or from the quoted wavelength (3 cm) we infer

$$k_x \sim 2.5 \text{ cm}^{-1} . \tag{7}$$

The authors do not state the density scale length but from their Fig. 2 and previous descriptions of the device[12,13] a reasonable inference is

$$L_n \sim 5 \text{ cm}. \tag{8}$$

So

$$(k_x L_n)^{-1} \sim 0.08, \tag{9}$$

whereas the measured fluctuation level was

$$\frac{\tilde{n}}{n_o} \sim 0.2. \qquad (10)$$

The discrepancy is not as significant in this case, but the remarkable result from this experiment is that, notwithstanding the high fluctuation level, no anomalous diffusion was observed! (More precisely, the transport estimated on the basis of quasi-linear theory and the observed fluctuation level is $\sim 10^3$ times greater than the observed upper bound to the anomalous transport.)

(c) Saison, Wimmel and Sardei[14] solved numerically in two dimensions the non-linear trapped-fluid equations used by Kadomtsev and Pogutse[15] as a description of the trapped-ion mode. They compared their numerically obtained values of the diffusion coefficient (at late times, when the instability saturates) with D_{KP} , the diffusion coefficient suggested by Kadomtsev and Pogutse for this instability. (D_{KP} is in fact equal to $\gamma_k/2k_y^2$. Linear theory does not provide a k_x. Kadomtsev and Pogutse[15] assume approximately isotropic turbulence, $k_x^2 \approx k_y^2 \approx k_\perp^2/2$, and are followed in this assumption by Düchs et al.[5].)

Although Saison et al. found that the density fluctuations in the steady state were an 'ordered pattern' rather than 'turbulence', the calculated diffusion coefficients were up to 20 times higher than D_{KP} and the scaling was Bohm-like.

II. Shear Stabilization of Drift Waves

A. Introduction

The basic theory of shear stabilization of drift waves is simple and physical[16-18]. If the magnetic field is in the z direction

and the density gradient is in the x direction, the simplest theory of drift waves gives, of course, $\omega = k_y v_D \equiv k_y c T_e / eBL_n$ $(n_o L_n^{-1} \equiv dn_o/dx)$, where the notation is standard. If one allows for ion inertia both perpendicular and parallel to the magnetic field, the dispersion relation becomes

$$- k_x \hat{\rho}_i^2 + \frac{k_z^2 v_s^2}{\omega^2} + \left(\frac{k_y v_D}{\omega} - 1 - k_y^2 \hat{\rho}_i^2 \right) = 0, \tag{11}$$

where $\hat{\rho}_i^2 \equiv T_e / M\omega_{ci}^2 \equiv v_s^2/\omega_{ci}^2$. In an inhomogeneous system the dispersion relation, Eq. (11), becomes a second order differential equation for the perturbed potential. If $x = 0$ is the point of maximum density gradient, then $k_x^2 = \frac{d^2}{dx^2}$, $k_z^2 = k_y^2 \frac{x^2}{L_s}$ (where L_s is the shear length) and $v_D = v_{DO} (1 - \frac{x^2}{\lambda_n^2})$, say. (Typically, $\lambda_n \sim L_n$.) The equation for φ then becomes

$$\hat{\rho}_i^2 \frac{d^2\varphi}{dx^2} + \left(\frac{v_s^2}{v_{DO}^2 L_s^2} - \frac{1}{\lambda_n^2} \right) x^2 \varphi$$
$$+ \left(\frac{k_y v_{DO}}{\omega - i\gamma} - 1 - k_y^2 \hat{\rho}_i^2 \right) \varphi = 0, \tag{12}$$

assuming $\omega \approx k_y v_{DO}$. Also we have assumed that some effect (i.e. inverse Landau damping) gives rise to a growth rate γ.

Equation (12) displays two fundamentally different types of behavior depending on the sign of $\left(\frac{v_s^2}{v_{DO}^2 L_s^2} - \frac{1}{\lambda_n^2} \right)$. If it is negative, localized modes can exist[17] and the dispersion relation is

$$\omega = i\gamma + \frac{k_y v_{DO}}{1 + k_y^2 \hat{\rho}_i^2 + \hat{\rho}_i \left(- \frac{v_s^2}{v_{DO}^2 L_s^2} + \frac{1}{\lambda_n^2} \right)^{\frac{1}{2}} \left(n + \frac{1}{2} \right)} \tag{13}$$

where n is an arbitrary integer. Thus as long as

$$\frac{v_s^2}{v_{DO}^2 L_s^2} < \frac{1}{\lambda_n^2} \quad ,$$

or equivalently

$$\frac{L_n^2}{\hat{\rho}_i^2 L_s^2} < \frac{1}{\lambda_n^2} \quad , \tag{14}$$

there is no stabilization arising from the shear.

On the other hand if the inequality[14] is reversed there are no localized modes. If some mechanism localized near $x = 0$ excites an instability, the proper eigenfunction is one which has outgoing energy flux at both $x = -\infty$ and $x = +\infty$. This eigenfunction then gives rise to the dispersion relation:

$$\omega = i\gamma + \frac{k_y v_{DO}}{1 + k_y^2 \hat{\rho}_i^2 + i\,\hat{\rho}_i \left(\frac{v_s^2}{v_{DO}^2 L_s^2} - \frac{1}{\lambda_n^2}\right)^{\frac{1}{2}} \left(n + \frac{1}{2}\right)} \tag{15}$$

$$\approx i \left\{ \gamma - \hat{\rho}_i \left(\frac{v_s^2}{v_{DO}^2 L_s^2} - \frac{1}{\lambda_n^2}\right)^{\frac{1}{2}} \left(n + \frac{1}{2}\right) k_y v_{DO} \right\} + k_y v_{DO} . \tag{16}$$

Thus the wave is now stabilized by the shear as long as

$$\gamma < \frac{1}{2} \left(\frac{v_s^2}{v_{DO}^2 L_s^2} - \frac{1}{\lambda_n^2}\right)^{\frac{1}{2}} \hat{\rho}_i k_y v_{DO} \quad . \tag{17}$$

Finally, even if there are no unstable eigenfunctions, it has been shown that drift waves can be convectively unstable and that increasing the shear can stabilize these convective instabilities by reducing the number of growth lengths in the unstable region[18].

Recent theory has diverged from this simple picture in two different ways.

(a) In toroidal geometry, variations in curvature drift terms may lead to substantial reduction or even complete elimination of shear damping[19,20] (Divergence A).

(b) In slab (or cylindrical) geometry, and in the absence of current along the field,[21,22] the electron response in the narrow region $\omega/k_{\parallel} > v_e$ renders all drift waves stable as long as $\frac{L_n}{L_s} > \frac{\hat{\rho}_i}{\lambda_n}$.

B. <u>The Evidence</u>

There are now at least two experiments[10,23] in toroidal geometry where divergence A might be expected to apply but which show shear stabilization of drift waves. In ref. 23 the fluctuations, which were identified as drift waves, were observed to be strongly excited, with amplitude independent of shear length, for $L_s > \lambda_n v_s/v_D$. As soon as L_s decreased to the point where the above inequality reverses, there occurred a steady decrease in fluctuation amplitude with increasing shear.

Secondly, in the experiment of Okabayashi and Arunasalam[10], generally $L_n/L_s \ll \hat{\rho}_i/\lambda_n$. These authors found that (i) increasing the shear led to a decrease in the fluctuations, (ii) the experimentally measured ion-mass dependence, and also the absolute magnitude, of the shear marginal stability condition was in reasonable agreement with the predictions of Pearlstein and Berk[16], and Rutherford and Frieman[18], and (iii) with high and moderate shear the unstable waves could be readily identified as drift waves.

In slab geometry there is at least one experiment[24] demonstrating shear stabilization. However here $L_n/L_s \sim \hat{\rho}_i/\lambda_n$, so it is difficult to know whether stabilization is due to the fact that electrons very near to the point at which $k_\parallel = 0$ stabilize the modes, or whether the more conventional theories are operative.

However, in the particle simulations of Lee, Kuo and Okuda[25] the density profile is initially exponential and therefore $\lambda_n = \infty$. The geometry is plane slab so that divergence B might be expected to apply. Nevertheless these authors definitely see growth of unstable drift waves even at fairly large values of shear. (Furthermore they have verified that there is no wave reflection, either from internal turning points or from external walls.) Also they state that the shear marginal stability condition is reasonably well approximated by the standard theory of Perlstein and Berk[16].

III. Conclusions

On the issue of nonlinear stabilization, recent work shows that fluctuation levels can be significantly higher than that suggested by Kadomtsev[4] and diffusion can be significantly larger than γ/k^2. Also there does not seem to be any simple general relation between diffusion and fluctuation levels.

On the other hand recent results show that, in spite of theoretical uncertainties, the simplest theory of shear stabilization works fairly well in both slab and toroidal geometry.

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